Heat-Kernel Approach to the Overlap Formalism *

Shoichi Ichinose

Department of Physics, University of Shizuoka
Yada 52-1, Shizuoka 422-8526, Japan.

February 1, 2008

We present a new regularization method, for d dim (Euclidean) quantum field theories in the continuum formalism, based on the domain wall configuration in (1+d) dim space-time. It is inspired by the recent progress in the chiral fermions on lattice. The wall "height" is given by $1/M$, where M is a regularization mass parameter and appears as a (1+d) dim Dirac fermion mass. The present approach gives a *thermodynamic view* to the domain wall or the overlap formalism in the lattice field theory. We will show qualitative correspondence between the present continuum results and those of lattice. The extra dimension is regarded as the (inverse) *temperature* t. The domains are defined by the *directions* of the "system evolvement", not by the sign of M as in the original overlap formalism. We take the 4 dim QED and 2 dim chiral gauge theory as examples. Especially the consistent and covariant anomalies are correctly obtained.

PACS. 11.30.Rd, 11.25.Db, 05.70.-a, 11.10.Kk, 11.10.Wx. -

I. Introduction

Regularizing quantum theories respecting the chirality has been a long-lasting problem both in the discrete and in the continuum field theories. The difficulty originates from the fact that the chiral symmetry is a symmetry strongly bound to the space-time dimension and is related to the discrete symmetry of parity and to the global features of the space-time topology. Non-continuous property is usually difficult to regularize. Ordinary regularizations, such as the dimensional regularization, often hinder controlling the chirality. The symmetry should be compared with others such as the gauge symmetry of the internal space and the Lorentz symmetry of the space-time. In the lattice field theory, the difficulty appears as the doubling problem of fermions[1] (see a text, say, [2]) and as the Nielsen-Ninomiya no-go theorem [3]. The recent very attractive progress in the lattice chiral fermion tells us the domain wall configuration in one dimension higher space(-time) serves as a good regularization, at least, as far as vector theories are concerned [4, 5]. It was formulated as the overlap formalism[6, 7] and

* Talk at CHIRAL'99,Taipei,Sep.13-18,1999; Univ. of Shizuoka preprint, US-99-09

was further examined by [8, 9, 10]. The corresponding lattice models were analyzed by [11, 12, 13, 14, 15]. The numerical data also look to support its validity[16]. Most recently the overlap Dirac operator by Neuberger[17], which satisfies Ginsparg-Wilson relation[18], and Lüscher's chiral symmetry on lattice[19] makes the present direction more and more attractive.

The overlap formalism has been newly formulated using the heat-kernel[20]. The heat-kernel formalism is most efficiently expressed in the coordinate space[21], and which enables us to do comparison with the lattice formalism. We will often compare the present results with those obtained by the lattice domain wall approach. Through the analysis we expect to clarify the essence of the regularization mechanism more transparently than on lattice. We see some advanced points over the ordinary regularizations in the continuum field theories. The main goal is to develop a new feasible regularization, in the continuum formalism, which is compatible with the chiral symmetry.

The present formalism is based on three key points:

1. We utilize the characteristic relation of heat and temperature, that is, heat propagates from the high temperature to the low temperature (the second law of thermodynamics). In the system which obey the heat equation, there exists a *fixed direction* in the system evolvement. We regard the heat equation for the spinor system, after the Wick rotation, as the Dirac equation in one dimension higher space-time. And we consider the (1+4 dim classical) configuration which has a fixed direction in time. This setting is suitable for regularizing the dynamics (in 4 dim Euclidean space) with control of the chirality.

2. Anti-commutativity between the system operator $\hat{D}$ and the chiral matrix, that is, $\gamma_5\hat{D} + \hat{D}\gamma_5 = 0$ plays the crucial role to separate the whole configuration into two parts (we will call them "(+)-domain" and "(-)-domain") which are related by the sign change of the "time"-axis. This is contrasted with the original formulation of the overlap where the difference of two vacua, one is constructed from the (+) sign regularization (1+4 dim fermion) mass and the other from the (-) sign, distinguishes the two domains.

3. Taking the small momentum region compared to the regularization mass scale M regularize the ultra-violet divergences and, at the same time, controls the chirality.

II. Domain Wall Regularization

Let us analyze the 4 dim massless Euclidean QED in the domain wall approach. First we express the effective action $\ln Z[A] = \mathrm{Tr}\ \ln \hat{D}$ in

terms of $\hat{D} = i\gamma_\mu(\partial_\mu + ieA_\mu)$ itself (not its square). Formally we have

$$\mathrm{Tr}\,\ln\hat{D} = -\mathrm{Tr}\int_0^\infty \frac{\mathrm{e}^{-t\hat{D}}}{t}dt = -\int_0^\infty \frac{dt}{t}\mathrm{Tr}\,[\frac{1}{2}(1+i\gamma_5)\mathrm{e}^{+it\gamma_5\hat{D}} + \frac{1}{2}(1-i\gamma_5)\mathrm{e}^{-it\gamma_5\hat{D}}]. \quad (1)$$

Because the eigenvalues of $\hat{D}$ are both negative and positive, the t-integral above is divergent. We clearly need regularization to make it meaningful. We should notice here that the final equality above relies only on the following properties of $\hat{D}$ and γ_5: $\gamma_5\hat{D} + \hat{D}\gamma_5 = 0$, $(\gamma_5)^2 = 1$. Note that, in the final expression of (1), the signs of the eigenvalues of $\hat{D}$ become less important for the t-integral convergence. This is because the *exponential* operator $\mathrm{e}^{-t\hat{D}}$ is replaced by the *oscillating* operators $\mathrm{e}^{\pm it\gamma_5\hat{D}}$ due to the above properties † . Here we introduce a regularization parameter M, which is most characteristic in this approach. M is taken to be positive for simplicity: $M > 0$.

$$\ln Z = \mathrm{Tr}\,\ln\hat{D} = -\lim_{M\to 0}\int_0^\infty \frac{dt}{t}\frac{1}{2}(1 - i\frac{\partial}{t\,\partial M})\mathrm{Tr}\,(G_+^{5M}(x,y;t) + G_-^{5M}(x,y;t)) \quad , \quad (2)$$

where $G_\pm^{5M}(x,y;t) \equiv < x|\exp\{\pm it\gamma_5(\hat{D}+iM)\}|y >$. ‡ M can be regarded as the "source" for γ_5. Through this procedure we can treat γ_5 within the new heat kernels $G_\pm^{5M}$. From its usage above, the limit $M \to 0$ should be taken in the following way before t-integral:

$$Mt \ll 1 \quad . \quad (3)$$

Very interestingly, the above heat-kernels satisfy the same 1+4 dim Minkowski Dirac equation after the following *Wick rotations* for t : $(i\partial\!\!\!/ - M)G_\pm^{5M} = ie A\!\!\!/ G_\pm^{5M}$, $(X^a) = (\mp it, x^\mu)$, where $A\!\!\!/ \equiv \gamma_\mu A_\mu(x)$, $\partial\!\!\!/ \equiv \Gamma^a\frac{\partial}{\partial X^a}$. ($\mu = 1,2,3,4;\ a = 0,1,2,3,4$) Note here that the sign of the Wick-rotation is different for G_+^{5M} and for G_-^{5M}. G_+^{5M} and G_-^{5M} turn out to correspond to (+)-domain and (-)-domain, respectively, in the original formulation [6, 7] and we also call them in the same way.

Both G_+^{5M} and G_-^{5M} are obtained in the same form G_M^5 specified by (G_0, S).

$$G_M^5(X,Y) = G_0(X,Y) + \int d^5Z\,S(X,Z)ie A\!\!\!/(z)G_M^5(Z,Y) \quad , \quad (4)$$

where $G_0(X,Y)$ is the free solution and $S(X,Z)$ is the propagator: $(i\partial\!\!\!/ - M)G_0(X,Y) = 0$, $(i\partial\!\!\!/ - M)S(X,Y) = \delta^5(X-Y)$. There are four choices of the propagator. (See Fig.1

† This statement is a disguise at the present stage. It is correct after the *Wick rotations* for t after (3).

‡ The case of $M = 0$ reduces to (1) where the relation $\gamma_5\hat{D} + \hat{D}\gamma_5 = 0$ is essential. In this sense we can consider this regularization procedure is related with the generalization of the above anti-commutation relation in some M-dependent way. It looks to correspond to a kind of Ginsparg-Wilson relation[18].

of [22].) From them we make three solutions (see Sec.III). They are obtained by some combinations of the positive and negative energy free solutions:

$$G_0^p(X,Y) \equiv -i\int \frac{d^4k}{(2\pi)^4}\Omega_+(k)\mathrm{e}^{-i\tilde{K}(X-Y)} \equiv \int \frac{d^4k}{(2\pi)^4}G_0^p(k)\mathrm{e}^{-ik(x-y)} \ ,$$
$$G_0^n(X,Y) \equiv -i\int \frac{d^4k}{(2\pi)^4}\Omega_-(k)\mathrm{e}^{+i\bar{K}(X-Y)} \equiv \int \frac{d^4k}{(2\pi)^4}G_0^n(k)\mathrm{e}^{-ik(x-y)} \ , \qquad (5)$$

where $\Omega_+(k) \equiv (M+\tilde{K}\!\!\!/)/2E(k)$, $G_0^p(k) \equiv -i\Omega_+(k)\mathrm{e}^{-iE(k)(X^0-Y^0)}$, $\Omega_-(k) \equiv (M-\bar{K}\!\!\!/)/2E(k)$, $G_0^n(k) \equiv -i\Omega_-(k)\mathrm{e}^{iE(k)(X^0-Y^0)}$, $E(k)=\sqrt{k^2+M^2}$, $(\tilde{K}^a)=(\tilde{K}^0=E(k),\ \tilde{K}^\mu=-k^\mu)$, $(\bar{K}^a)=(\bar{K}^0=E(k),\bar{K}^\mu=k^\mu)$. k^μ is the momentum in the 4 dim Euclidean space. $\tilde{K}$ and $\bar{K}$ are on-shell momenta($\tilde{K}^2=\bar{K}^2=M^2$), which correspond to the positive and negative energy states respectively. We can rewrite $\Omega_\pm(k)$ as

$$\Omega_+(k)=\frac{1}{2}(\gamma_5+\frac{M+i\,k\!\!\!/}{|M+i\,k\!\!\!/|}) \quad , \quad \Omega_-(k)=\frac{1}{2}(-\gamma_5+\frac{M+i\,k\!\!\!/}{|M+i\,k\!\!\!/|}) \qquad (6)$$

where $|M+i\,k\!\!\!/| \equiv E(k)$. The factor $\frac{M+i\,k\!\!\!/}{|M+i\,k\!\!\!/|}$ can be regarded as a "phase" operator *depending on configuration*. The expressions above look similar to the overlap Dirac operator [17] for the case of *no* Wilson term. § In fact, $\Omega_\pm(k)$ have *projective property* with their hermite conjugate. The relations will be efficiently used in anomaly calculations.

The final important stage is regularization of the (1-loop) ultraviolet divergences. Corresponding to the 1-loop quantum evaluation, the determinant (1) finally involves one momentum(k^μ)-integral (besides t-integral). We will take the analytic continuation method in order to avoid introducing further regularization parameters and to avoid breaking the gauge invariance. It can be shown[22] that the method is essentially equivalent to restricting the integral region from $0 \leq |k^\mu| < \infty$ to

$$\text{Chiral Condition} \ : \ 0 \leq |k^\mu| \leq M \quad . \qquad (7)$$

¶ This looks similar to the usual Pauli-Villars procedure in the point of ultra-violet regularization. M plays the role of the momentum cut-off. We should stress that this

---

§ The overlap Dirac operator D on lattice is $S_F = a^4\sum_{x\in\text{all sites}}\bar{\psi}(x)D\psi(x),\ aD = 1+\gamma_5\frac{H}{\sqrt{H^2}}$, $\gamma_5 H = \sum_\mu\{\frac{1}{2}\gamma_\mu(\nabla_\mu+\nabla^*_\mu)-\frac{a}{2}\nabla^*_\mu\nabla_\mu\}-M$, where a is a lattice spacing. If we ignore the $\nabla^*_\mu\nabla_\mu$-term (Wilson term), the form is quite similar to (6).

¶ Instead of the analytic continuation, we can take the higher derivative regularization. This corresponds to the Wilson term in lattice: $i\,\partial\!\!\!/ - M \ \rightarrow \ i\,\partial\!\!\!/ \pm \frac{r}{M}\partial^2 - M$, r: "Wilson term" coefficient. In this case the unitarity problem, rather than the gauge invariance, should be clarified.

restriction condition (7) on the momentum integral, at the same time, controls the chirality as explained in the following. (This point is a distinguished property of the domain wall regularization.) We call (7) *chiral condition*. Its *extreme* case:

$$\text{Extreme Chiral Limit}\ :\ \frac{M}{|k^\mu|}\rightarrow\infty\quad , \tag{8}$$

implies the chirality selection:

$$\begin{aligned}&\text{for (+)-domain } (X^0=-it)\quad ,iG_0^p(k)\rightarrow\frac{1+\gamma_5}{2}\mathrm{e}^{-Mt}\ ,\ iG_0^n(k)\rightarrow\frac{1-\gamma_5}{2}\mathrm{e}^{+Mt}\ ;\\ &\text{for (-)-domain } (X^0=+it)\quad ,iG_0^p(k)\rightarrow\frac{1+\gamma_5}{2}\mathrm{e}^{+Mt}\ ,\ iG_0^n(k)\rightarrow\frac{1-\gamma_5}{2}\mathrm{e}^{-Mt}\ .\end{aligned} \tag{9}$$

This result will be used for characterizing different configurations with respect to the chirality. We use (7) instead of (8) in the concrete calculation. (8) is *too restrictive* to keep the dynamics. Loosening the extreme chiral limit (8) to the chiral condition (7) can be regarded as a part of the present regularization. This situation looks similar to the introduction of the Wilson term, in the lattice formalism, in order to break the chiral symmetry.

Let us reexamine the condition (3). As read from the above result, the domain is characterized by the exponential damping behavior which has the "width"$\sim 1/M$ around the origin of the extra t-axis. (3) restricts the region of t as $t\ll 1/M$. This is for considering only the massless mode as purely as possible. In the lattice formalism, this corresponds to taking the zero mode (surface state) limit, in order to avoid the doubling problem, by introducing many "flavor" fermions (or adding an extra dimension) and many bosonic Pauli-Villars fields to kill the heavy fermions contribution. Besides the extreme chiral limit (8), we often consider , corresponding to (3), the following limit:

$$M|X^0-Y^0|\rightarrow+0\quad . \tag{10}$$

This limit will be taken to characterize the full solutions (4) by their *boundary conditions*.

In the lattice numerical simulation, the best fit value of the regularization mass M looks restricted both from the below and from the above depending on the simulation "environment" [23, 16]. [‖] (M $\sim$a few Gev for the hadron simulation.) The similar one occurs in the present regularization. The "double" limits (3) and (7) or (8) imply

$$|k^\mu|\ll M\ll\frac{1}{t}\quad\text{or}\quad|k^\mu|\leq M\ll\frac{1}{t}\quad . \tag{11}$$

[‖] In the lattice formalism the corresponding bound on M has been known, from the requirement of no doublers, since the original works[4, 6].

In the standpoint of the extra dimension, the limit $M \ll \frac{1}{t}$ $(Mt \to +0)$ corresponds to , combined with the condition on $|k^\mu|/M$, taking the dimensional reduction from 1+4 dim to 4 dim (Domain wall picture of 4 dim space). The relation (11) is the most characteristic one of the present regularization. It should be compared with the usual heat-kernel regularization where only the limit $t \to +0$ is taken and the ultraviolet regularization is done by the simple subtraction of divergences. Eq.(11) shows the delicacy in taking the limit in the the present 1+4 dimensional regularization scheme. It implies, in the lattice simulation, M should be appropriately chosen depending on the regularization scale (,say, lattice size) and the momentum-region of 4 dim fermions.

III. (+)-Domain and (-)-Domain ($G^{5M}_\pm$) III-1. Feynman Path and Anti-Feynman Path

First we consider the Feynman propagator: $S_F(X,Y) = \theta(X^0 - Y^0)G_0^p(X,Y) + \theta(Y^0 - X^0)G_0^n(X,Y)$. It has both the retarded and advanced parts. Now we remind ourselves of the fact that there exists a fixed direction in the system evolvement when the temperature parameter works well. Let us regard the extra axis, after the Wick-rotations, as a temperature. Assuming the analogy holds here, we try to adopt the following *directed* solution displayed in TABLE I.

	G^{5M}_+(Retarded)	G^{5M}_-(Advanced)
G_0	$G_0^p(X,Y)$	$G_0^n(X,Y)$
S	$\theta(X^0 - Y^0)G_0^p(X,Y)$	$\theta(Y^0 - X^0)G_0^n(X,Y)$
F.E.	$(i\hat{\partial} - M)G^{5M}_+ = ie\frac{1+\gamma_5}{2}\not{A}G^{5M}_+ + O(\frac{1}{M})$	$(i\hat{\partial} - M)G^{5M}_- = ie\frac{1-\gamma_5}{2}\not{A}G^{5M}_- + O(\frac{1}{M})$

TABLE I. (+)-domain and (-)-domain in the Feynman path solution.

This is chosen in such a way that the t-integral converges. Because we have "divided" a full solution into two chiral parts in order to introduce a *fixed direction* in the system evolvement, $G^{5M}_\pm$ defined in TABLE I do not satisfy $(i\hat{\partial} - M)G^5_M = ie\not{A}G^5_M$ but satisfy , in the extreme chiral limit, the field eq. of the *chiral* QED: $\hat{D}_\pm \equiv i(\not{\partial} + ie\frac{1\pm\gamma_5}{2}\not{A})$. Taking the extreme chiral limit in the momentum spectrum, we can read off the domain wall structure near the origin of the extra axis. (See Fig.2 of [22].) The full solutions $G^{5M}_\pm$ (4) of TABLE I satisfy the boundary condition :

$$
\begin{aligned}
i(G^{5M}_+(X,Y) - G^{5M}_-(X,Y)) &\to \gamma_5\delta^4(x-y) \text{ as } M|X^0 - Y^0| \to +0 \ , \\
i(G^{5M}_+(X,Y) + G^{5M}_-(X,Y)) &\to \int \frac{d^4k}{(2\pi)^4}\frac{M + i\not{k}}{|M + i\not{k}|}\mathrm{e}^{-ik(x-y)} \text{ as } M|X^0 - Y^0| \to +0 \ .(12)
\end{aligned}
$$

Taking into account the boundary conditions above, we should take, in the Adler-Bell-Jackiw and Weyl anomaly calculations, as

$$
\frac{1}{2}\delta_\alpha \ln J_{ABJ} = \lim_{M|X^0 - Y^0| \to +0} \mathrm{Tr}\ i\alpha(x) i(G^{5M}_+(X,Y) - G^{5M}_-(X,Y)) \quad ,
$$

$$\frac{1}{2}\delta_\omega \ln J_W = \lim_{M|X^0-Y^0|\to+0} \mathrm{Tr}\ \omega(x) i\gamma_5 (G_+^{5M}(X,Y) - G_-^{5M}(X,Y)) \quad . \tag{13}$$

The meaning of the choice of Feynman path solution (TABLE I) is subtle (but interesting), because the solution does not satisfy $(i\partial\!\!\!/ - M)G_M^5 = ie A\!\!\!/ G_M^5$. The clear separation of right and left and its calculational simplicity fascinate us to examine this solution.

We can take the opposite choice of G_0^p and G_0^n in TABLE I. We call this case anti-Feynman path solution. The regularization using this solution turns out to give the same result as the Feynman path solution. The different point is that, due to the presence of the exponentially growing factor $\mathrm{e}^{+E(k)t}$, we must do calculation in the X^0-coordinate.[20]

III-2. Symmetric Path

Let us consider the symmetric pathes. In this case we are led to take the solution displayed in TABLE II.

	G_+^{5M}(Retarded)	G_-^{5M}(Advanced)
G_0	$G_0^p(X,Y) - G_0^n(X,Y)$	$G_0^n(X,Y) - G_0^p(X,Y)$
S	$\theta(X^0-Y^0)(G_0^p(X,Y) - G_0^n(X,Y))$	$\theta(Y^0-X^0)(G_0^n(X,Y) - G_0^p(X,Y))$
F.E.	$(i\partial\!\!\!/ - M)G_+^{5M} = ie A\!\!\!/ G_+^{5M}$	$(i\partial\!\!\!/ - M)G_-^{5M} = ie A\!\!\!/ G_-^{5M}$

TABLE II. (+)-domain and (-)-domain in the symmetric path solution.

$G_\pm^{5M}$ satify the QED field equation properly. Taking the extreme chiral limit $\frac{|k^\mu|}{M} \ll 1$ in the solution of TABLE II, we can read off the symmetric wall structure (one wall at the origin and the other at the infinity, see Fig.4 of [22]). The above solutions satisfy the following boundary condition:

$$\frac{i}{2}(G_+^{5M}(X,Y) - G_-^{5M}(X,Y)) \to \gamma_5 \delta^4(x-y) \text{ as } M|X^0-Y^0| \to +0 \ . \tag{14}$$

In this case, the anomalies are regularized as

$$\delta_\alpha \ln J_{ABJ} = \lim_{M|X^0-Y^0|\to+0} \mathrm{Tr}\ i^2\alpha(x)\{G_+^{5M}(X,Y) - G_-^{5M}(X,Y)\} \quad ,$$
$$\delta_\omega \ln J_W = \lim_{M|X^0-Y^0|\to+0} \mathrm{Tr}\ i\omega(x)\gamma_5\{G_+^{5M}(X,Y) - G_-^{5M}(X,Y)\} \quad . \tag{15}$$

Both in (13) and in (15), the anomalies are expressed by the "difference" between G_+^{5M} and G_-^{5M} contributions. This exactly corresponds to the "overlap" equation in the original formalism. The "difference" in the effective action corresponds to the "product" in the partition function between (+) part and (-) part, that is the "overlap". This is the reason we name $G_\pm^{5M}$ as ($\pm$)-domains.

IV. Anomaly Calculations

Using the anomaly equations in Sec.III, explicit calculation has been done for 2 dim QED, 4 dim QED and 2 dim chiral gauge theory. As for

the former two theories, it is confirmed that the anomalies are correctly obtained in the symmetric path solution, whereas $(1/2)^{d/2}$ times of them for the Feynman path solution. As for the last theory, the consistent and covariant anomalies are correctly obtained by taking the chiral vertex for the former and the hermitian vertex for the latter (with the common choice of the symmetric path solution). See [20, 22, 24] for details.

IV. Discussion and Conclusion

From the results of Sec.III, we can imagine that the choice of (anti-)Feynman path solution perturbatively defines the chiral version of the original theory, for example, the chiral gauge theory. As far as anomaly calculation is concerned, it holds true. In order to show the statement definitely, we must clarify the following things. The "ordinary" chiral symmetry appears only in the limit : $\frac{|k^\mu|}{M} \rightarrow +0$. But this limit can not be taken because it "freeze" the dynamics and anomalies do *not* appear. It seems we must introduce some new "softened" version of the chiral symmetry which keeps the dynamics. One standpoint taken in this paper is to replace $|k^\mu|/M \ll 1$ by $|k^\mu|/M \leq 1$. It breaks the "ordinary" chiral symmetry. It could, however, be possible that this replacement can avoid the breaking by changing (generalizing) the "ordinary" chiral symmetry. In this case, the new chiral Lagrangian has infinitely many higher-derivative terms. We should explain this new "deformed" Lagrangian from some generalized chiral symmetry. This situation looks similar to what Lüsher[19] did for the chiral lattice.

The chiral problem itself does not depend on the interaction. It looks a kinematical problem in the quantization of fields. How do we treat the different propagations of *free* solutions depending on the boundary conditions (with respect to the Wick-rotated time) is crucial to the problem. In the standpoint of the operator formalism (the Fock-space formalism) it corresponds to how to treat the "delicate" structure (due to the ambiguity of the fermion mass sign) of the vacuum of the free fermion theory. The present paper insists the following prescription: First we go to 1+4 dim Minkowski space by the Wick-rotation of the inverse temperature t, and take the "directed" solution. The anomaly phenomena concretely reveal the chiral problem. The proposed prescription passes the anomaly test.

References

[1] K.G.Wilson, New phenomena in sub-nuclear physics (Erice,1975), ed. A.Zichichi (Plenum, New York,1977),Part A,69

[2] M.Creutz,"Quarks,gluons and lattices", Cambridge Univ. Press, Cambridge, 1983

[3] H.Nielsen and M.Ninomiya, Nucl.Phys.**B185**(1981)20; **B193**(1981)173

[4] D.B.Kaplan,Phys.Lett.**B288**(1992)342

[5] K.Jansen,Phys.Lett.**B288**(1992)348

[6] R.Narayanan and H.Neuberger,Nucl.Phys.**B412**(1994)574;Phys.Rev.Lett.**71**(1993)3251

[7] R.Narayanan and H.Neuberger,Nucl.Phys.**B443**(1995)305

[8] Randjbar-Daemi and J.Strathdee,Phys.Lett.**B348**(1995)543

[9] Randjbar-Daemi and J.Strathdee,Nucl.Phys.**B443**(1995)386

[10] Randjbar-Daemi and J.Strathdee,Phys.Lett.**B402**(1997)134

[11] Y.Shamir,Nucl.Phys.**B406**(1993)90

[12] M.Creutz and J.Horváth,Phys.Rev.**D50**(1994)2297

[13] M.Creutz,Nucl.Phys.B(Proc.Suppl.)**42**(1995)56

[14] V.Furman and Y.Shamir,Nucl.Phys.**B439**(1995)54

[15] P.M.Vranas,Phys.Rev.**D57**(1998)1415

[16] P.Vranas et al., hep-lat/9903024,"Dynamical lattice QCD thermodynamics and the $\mathrm{U}(1)_A$ symmetry with domain wall fermions"

[17] H.Neuberger,Phys.Lett.**B417**(1998)141

[18] P.H.Ginsparg and K.G.Wilson, Phys.Rev.**D25**(1982)2649

[19] M.Lüscher, Phys.Lett.**B428**(1998)342

[20] S.Ichinose, US-98-09, hep-th/9811094, to be published in Phys.Rev.D, "Temperature in Fermion Systems and the Chiral Fermion Determinant"

[21] S.Ichinose and N.Ikeda, Jour.Math.Phys.**40**(1999)2259

[22] S.Ichinose,Univ.of Shizuoka preprint, US-99-02, hep-th/9908156, "New Regularization Using Domain Wall"

[23] T.Blum and A.Soni,Phys.Rev.**D56**(1997)174; Phys.Rev.Lett.**19**(1997)3595

[24] S.Ichinose,AEI-1999-35,hep-th/9911079, "Renormalization using Domain Wall Regularization"